UPDATED LAGRANGIAN METHOD

CODE_BRIGHT TEAM

2020

In a standard finite element program, calculations are carried out with respect to the original geometry. The geometry is fixed and remains in the same position regardless of the displacements that occur. In a program, the fact that geometry is fixed, permits to calculate and store some geometrical quantities to be used for all time steps (like volumes of elements or any quantity derived from shape functions).

The idea of the Updated Lagrangian Method is that the geometry is updated after calculation of each time step. Coordinates of all nodes are recalculated using the position at the beginning of the time step and the increment of displacement during the time step for each node. This calculation is carried out after each time step is solved. A direct implication of the modification of the geometry is that the geometric quantities mentioned above should be recalculated because the shape, size and position of the elements changed.

The Updated Lagrangian Method is consistent with the fact that conservation equations are implemented in CODE_BRIGHT in its Lagrangian form (i.e. using the material derivative).

The Updated Lagrangian Method is considered a good approximation under the following conditions:

- The solution of the problem is made in small time steps, so the variations of geometry at each time step are small. This happens for nonlinear constitutive models and coupled problems.
- Small strain can be assumed for the constitutive behavior and for the finite element formulation. This is accomplished if the time steps are small.
- As time steps are small, small increments of displacements take place.
- Small rotations take place during the movement.
- The shape of all finite elements remains acceptable. So the finite elements do not distort too much.

The implementation in CODE_BRIGHT of the Updated Lagrangian Method includes the following steps. At the beginning of each time step the following tasks are done:

- Positions of the nodes are updated by adding the displacement vectors computed during the previous time step
- Geometrical variables calculated with the interpolation functions are recalculated (for instance element volume or surface)
- Initial values of the state variables at the updated nodes and integration points are the same as the final values of the state variables computed during the previous time step
- Body forces for the new time step are calculated with the new geometrical configuration
- Boundary conditions of stress or flux are calculated with the new geometrical configuration

The Updated Lagrangian Method is, therefore, a method for large displacements but for small deformations.

A couple of examples are presented in this document.

Example 1. Tunnel in rock salt

In rock salt formations, mining activities are difficult because cavities can close in decades. This is due to creep of rock salt, which depends on the type of rock (Halite, Silvite, etc), the stress level, temperature and humidity content. When the depth increases, both stress and temperature increase leading to higher convergence rates. In potash mines, drifts can close completely in less than 100 years. This is a quite fast convergence.

Figure 1.1 shows a model to calculate convergence of a 6 m diameter tunnel excavated in rock salt at a depth of 550 m. The model simulates an excavation and the long term deformation of an infinite tunnel in a salt rock medium. Elastic deformation is small compared with the creep long term deformations.

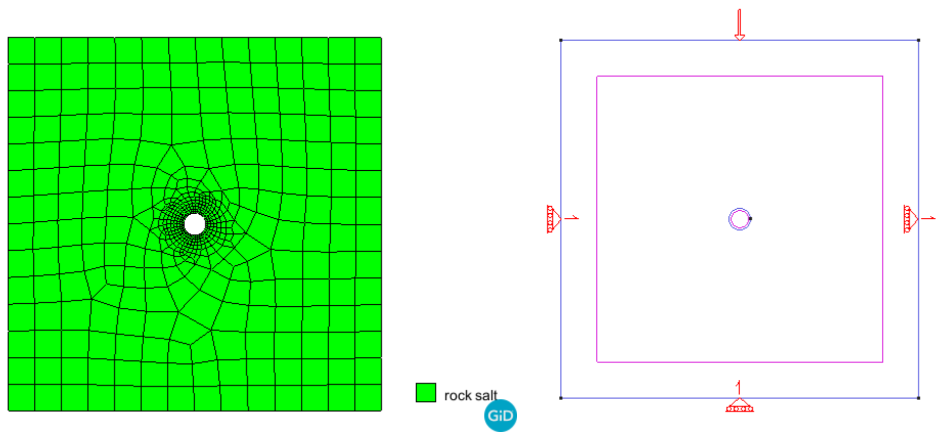

Figure 1.1 Circular tunnel with a diameter of 6 m in a homogeneous rock salt at a depth of 550 m. The domain is 100 by 100 m.

The results in figure 1.2 show that convergence rate is constant for the case of fixed geometry. This is because the calculation reaches a steady state with constant stress distribution and constant strain rate of deformation. No hardening or softening is considered by the constitutive model, only secondary or steady state creep. The fact that displacement grows linearly with time leads to the possibility that displacement becomes bigger than the radius of the tunnel, and this is not possible in reality.

In case the Updated Lagrangian Method is used, the convergence rate decreases as convergence increases. This is expected as the tunnel internal diameter is smaller. In the limit, the total closure of the tunnel corresponds to zero diameter and zero convergence rate. This is a process that is realistic and observed in salt rock formations.

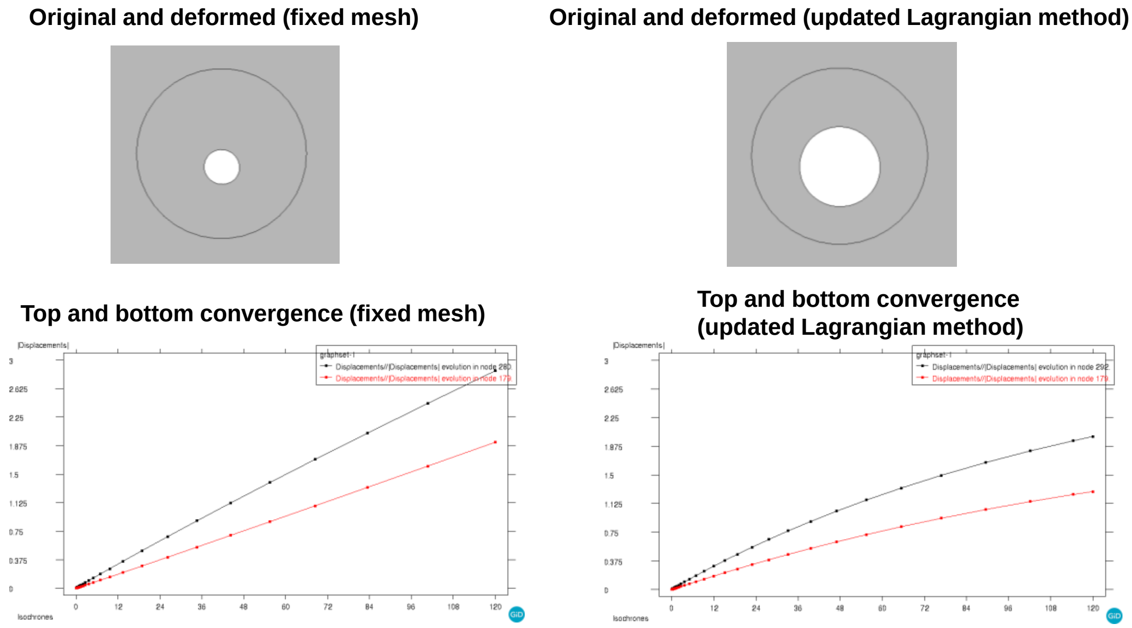

Figure 2.2 Long term convergence of a tunnel in rock salt using a fixed mesh (left) or an updated mesh (right). Note that the convergence rate decreases due to nonlinear geometry of the problem. Deformed geometry is represented with no amplification factor. Time in years.

Example 1. Wall under sliding failure

A wall under sliding failure is another example of the use of the Updated Lagrangian Method included in this document.

The objective of a numerical calculation in the case of retaining walls is usually to obtain a safety factor for failure. A finite element program can be used simply by application of the strength reduction method. The strength reduction method can be used in CODE_BRIGHT by setting a series of intervals where a strength parameter is reduced. Displacement can be set for each interval. A safety factor can be obtained by doing the quotient between the actual strength (for instance, tangent of friction angle) and the strength that produces failure. Failure can be defined by the case for which displacements increase very rapidly. This methodology is typical to study the stability of geotechnical structures.

Here, the Updated Lagrangian Method is used to illustrate that results during failure are different if the changes in geometry are considered. However, this type of calculations may have little practical interest when the stability of a retaining wall is analyzed.

A typical retaining wall in a granular soil (figure 2.1) may undergo failure by sliding if the strength of the soil is low. The model consists in a first interval in which equilibrium is achieved as the strength is sufficiently high. A viscoplastic model is used to represent failure because it permits to reproduce steady failure with continuation of the movement along time. Plasticity would lead to the development of displacements which tend to infinity. As viscoplasticity is considered, an arbitrary time scale can be used for an arbitrary viscosity (or fluidity: inverse of viscosity). The smaller the viscosity considered, the faster the displacement during failure. After stable equilibrium is achieved, the strength is reduced, and failure progresses.

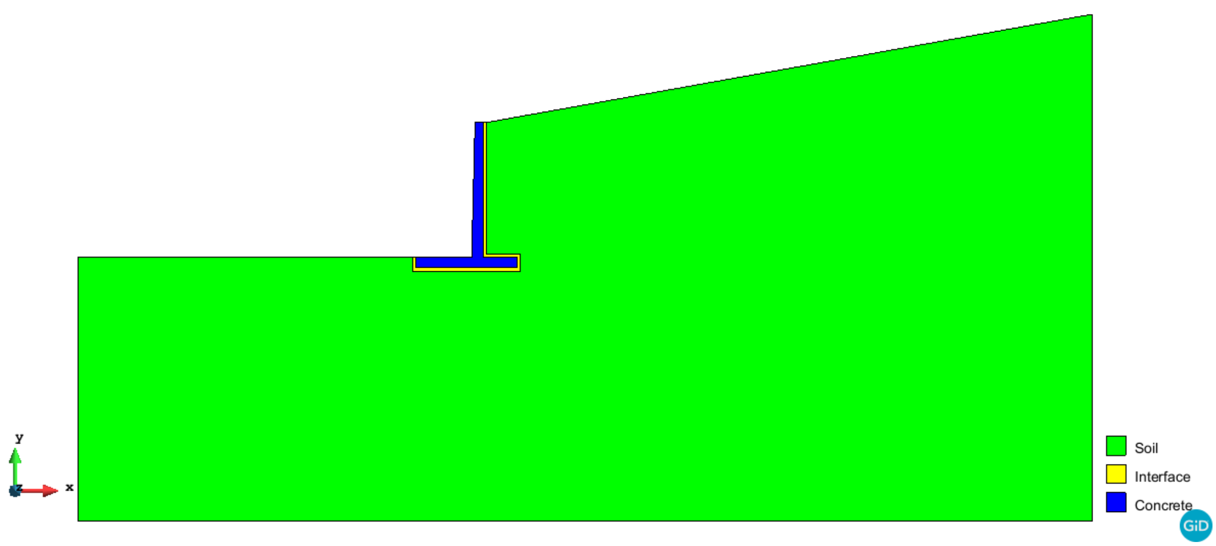

Figure 2.1. Retaining wall in a granular material (failure critical state slope q / p' = M = 0.8) including a 10 cm interface (reduction factor on M of R = 2/3). The difference between the bottom surface and the upper surface is 4 m, and the concrete is 0.25 m (top) to 0.30 m (bottom) thick.

Note that in this case there are not external forces or displacements which can be applied incrementally. The gravity body force is the driving force for failure.

Results are shown in Figure 2.2. It can be observed that failure leads to a constant velocity of the wall in the case the geometry is maintained. Even if the displacements become large, the velocity is the same, thus leading to infinite displacement if the calculation continues. In a real stability calculation (using for instance the reduction of strength method), displacements post failure are not realistic as they tend to infinity. So for a fixed geometry calculation, displacements post-failure do not represent reality (except in case that hardening occurs).

In case of geometry update, the displacement velocity vanishes up to a point in which the wall tends to stop. This is caused by the change in the geometry, stress distribution and equivalent forces.

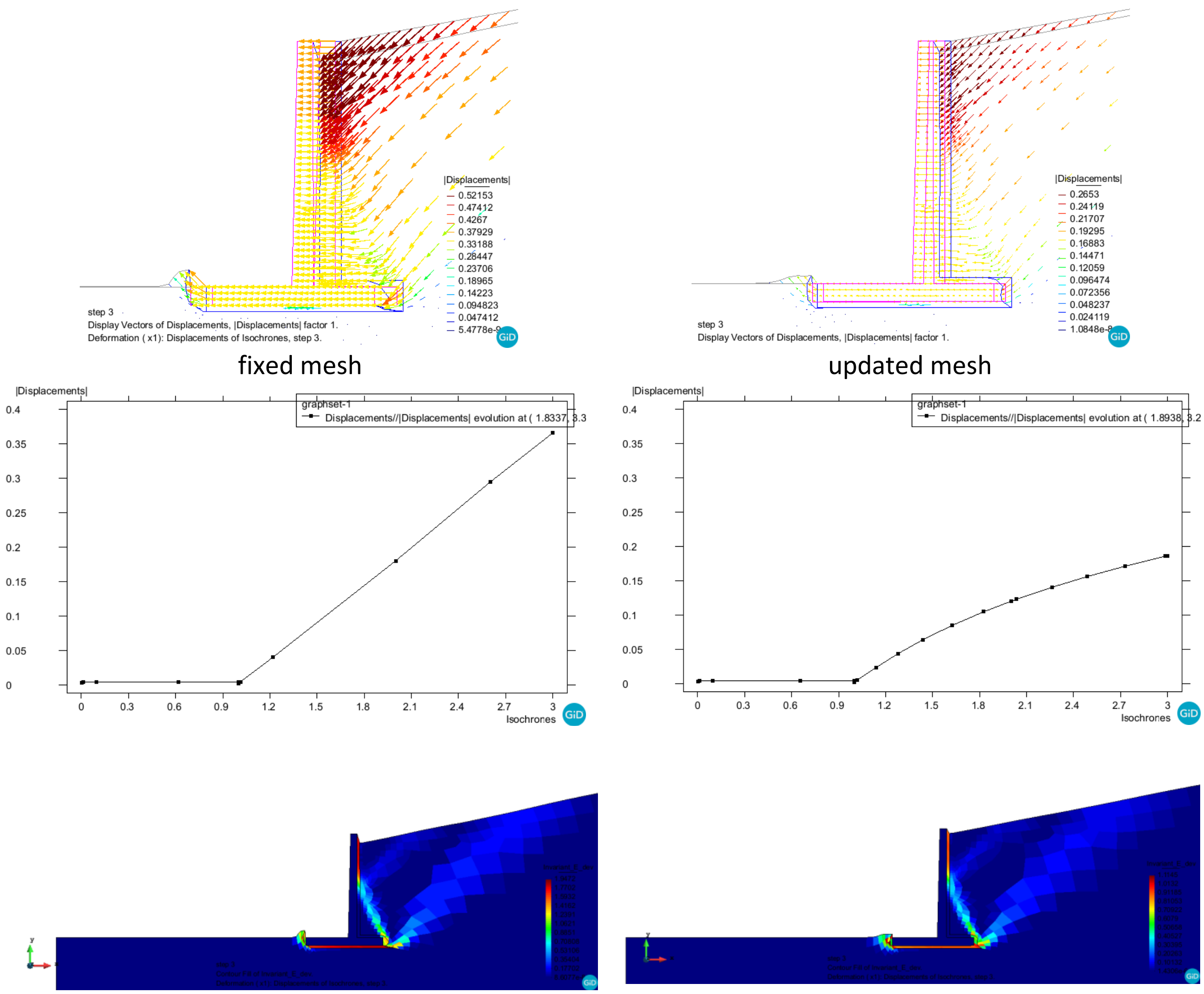

Figure 1.1. Failure of a retaining wall in a frictional soil. Displacement vectors, movement evolution and contour field of deviatoric strains.

Concluding remarks

The Updated Lagrangian Method is a first approximation to the large displacement analysis. The method can be described as a method for large displacements but for small strains and small rotations.

It simple takes the nodal point positions and updates the coordinates using the increment of displacements. The method implies a certain overload of calculations, as compared with the fixed mesh classical finite element formulation, because all the geometrical quantities in the finite element approximation should be recalculated, such as shape functions and derived quantities.

The Updated Lagrangian Method introduces the nonlinearity induced by the geometry variation in problems where large displacements occur. The method should be good in case the nonlinearity or the coupled nature of the problem leads to the necessity to calculated using a discretization in time sufficient so strains and rotations are considered small for each time step.

In this document, the method in applied to a tunnel in rock salt and to a retaining wall in a granular material. Large displacements occur in these problems. For fixed geometry, the models predict a constant displacement velocity. As the geometry is updated, displacement velocity reduces as movement progresses, leading to a situation in which movements stop.