INTERPRETATION OF WIND COMPONENTS AS COMPOSITIONAL VARIABLES

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Abstract

The classical statistical study of the wind speed in the atmospheric surface layer is made generally from the analysis of the three habitual components that perform the wind data, that is, the component W-E, the component S-N and the vertical component, considering these components independent.

When the goal of the study of these data is the Aeolian energy, so is when wind is studied from an energetic point of view and the squares of wind components can be considered as compositional variables. To do so, each component has to be divided by the module of the corresponding vector.

In this work the theoretical analysis of the components of the wind as compositional data is presented and also the conclusions that can be obtained from the point of view of the practical applications as well as those that can be derived from the application of this technique in different conditions of weather.

1. Introduction

In the atmospheric surface layer the behaviour of wind is generally studied starting from data related to its three components. If we denote by v_x the west-east component of wind data, by v_y to the south-north component and v_z to the vertical component, we can define a vector with those three components. This is the wind vector $\vec{v} = (v_x, v_y, v_z)$. In micrometeorology, these coordinates are usually denoted as *u, v, w*. This paper analyses the possibility of studying the wind components as compositional variables, when the squares of wind components are considered. This results when wind is studied from an energetic point of view. Transformations of compositional data in this case are also studied.

We recall that we call an *observation of m components* any element of the set

$$\mathbb{R}_+^m = \left\{ x = (x_1, x_2, \ldots, x_m) \in \mathbb{R}^m;\ x_j > 0,\ j = 1, 2, \ldots, m \right\}$$

We call *composition of m components* any element of the subset $S_m \subset \mathbb{R}_+^m$ defined by

$$S_m = \left\{ x \in \mathbb{R}_+^m ; x_1 + x_2 + \ldots + x_m = C \right\} \subset \mathbb{R}_+^m$$

In the above definition C is a constant named *closure constant*. The value of the closure constant depends on the units of data. For example, $C = 100$ if data are percentages, $C = 10^6$ if data are part per million (ppm), $C = 1$ if data are parts of the unit, etc. Because it is verified that $\sum_{j=1}^{m} x_j = C$, then $\sum_{j=1}^{m} \left(\frac{x_j}{C} \right) = 1$; so, for the sake of simplicity, we can assume that $C = 1$.

2. The wind components as compositional data

In the study of the dynamics in the atmospheric surface layer it is important to register the wind in three orthogonal directions. Consequently, the wind is expressed through three coordinates, two in the horizontal plane and the third one orthogonal to this plane. As a matter of convenience, in the horizontal plane, the parallel coordinate v_x is defined according in a west-east direction to the equator. The perpendicular coordinate v_y in a south-north direction and the coordinate v_z perpendicular to both coordinates increasing from the floor. These coordinates are considered forming a trihedron positively guided. So, according to these definitions, we can consider the wind vector as $\vec{v} = (v_x, v_y, v_z)$; we call it simply, the wind.

The Euclidean norm or the module of the wind, is given by the expression:

$$|\vec{v}| = v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

Considering the quotient between the wind and its module, we obtain the $\vec{u}$ vector as:

$$\vec{u} = \frac{1}{v} \vec{v} = \frac{1}{v} \left(v_x, v_y, v_z \right)$$

According to its definition, it will be a unit vector, that is

$$|\vec{u}| = 1$$

It is also true that

$$|\vec{u}|^2 = 1$$

Having done the above exposition, we can arrive to the compositional variable of the wind presented in the following section.

3. The wind kinetic coefficient

We define the wind kinetic coefficient, that we denote by $\vec{E}$, as the vector:

$$\vec{E} = (E_1, E_2, E_3) = \left(\left(\frac{v_x}{v}\right)^2, \left(\frac{v_y}{v}\right)^2, \left(\frac{v_z}{v}\right)^2 \right)$$

This vector is composed by three components; each one of them has no dimension (in physic sense). The vector $\vec{E}$ is related to the proportion of kinetic energy associated to the wind in each one of its directions. Also, for their definition, the vector $\vec{E}$ is such that its components are compositional variables, because the sum of these is 1. In fact:

$$E_1 + E_2 + E_3 = \left(\frac{v_x}{v}\right)^2 + \left(\frac{v_y}{v}\right)^2 + \left(\frac{v_z}{v}\right)^2 = \frac{v_x^2 + v_y^2 + v_z^2}{v^2} = \frac{v^2}{v^2} = 1$$

In the figure 1 we present an example of ternary diagram of the kinetic coefficient of the wind.

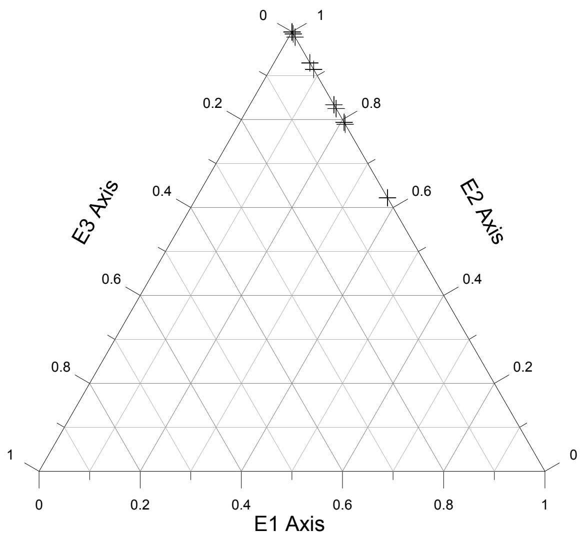

Figure 1: Example of ternary diagram of the kinetic coefficient of the wind

By this way we have been able to associate a compositional component to the wind vector. It is sought to study its applicational possibilities to the dynamic characteristics of the atmospheric surface layer.

The mathematical and statistical analysis of the compositional variables is usually made by a previous transformation applied to data. In the following section we present some examples of these.

4. Transformations of the wind kinetic coefficient

The transformations more commonly applied to compositional data values are: the additive logratio transformation (ALR), the centred logratio transformation (CLR), both defined by Aitchison (1986), and the most recent the isometric logratio transformation (ILR) defined by Egozcue et al. (2003).

In this section we present how these transformations are applied to the kinetic coefficient.

4.1 Additive logratio transformation *ALR*

The ALR transformation is defined as:

$$\begin{aligned} ALR : S_{n+1} &\to \mathbb{R}^n \\ (x_1, x_2, ..., x_{n+1}) &\to \left(\ln\frac{x_1}{x_{n+1}}, \ln\frac{x_2}{x_{n+1}}, ..., \ln\frac{x_n}{x_{n+1}} \right) \end{aligned}$$

In the case of the kinetic coefficient the application is:

$$\begin{aligned} ALR : S_3 &\to \mathbb{R}^2 \\ \vec{E} &\to ALR(\vec{E}) = \left(\ln\frac{E_1}{E_3}, \ln\frac{E_2}{E_3} \right) = (A_1, A_2) \end{aligned}$$

Then,

$$ALR(\vec{E}) = \left(\ln\frac{E_1}{E_3}, \ln\frac{E_2}{E_3} \right) = \left(\ln\frac{\left({}^{v_x}\!/\!{}_{v} \right)^2}{\left({}^{v_z}\!/\!{}_{v} \right)^2}, \ln\frac{\left({}^{v_y}\!/\!{}_{v} \right)^2}{\left({}^{v_z}\!/\!{}_{v} \right)^2} \right)$$

Simplifying

$$ALR(\vec{E}) = \left(\ln\frac{v_x^2}{v_z^2}, \ln\frac{v_y^2}{v_z^2} \right) = \left(\ln\left(\frac{v_x}{v_z} \right)^2, \ln\left(\frac{v_y}{v_z} \right)^2 \right)$$

By logarithm rules

$$ALR(\vec{E}) = \left(2(\ln|v_x| - \ln|v_z|), 2(\ln|v_y| - \ln|v_z|) \right)$$

In consequence the (A_1, A_2) components of the transformation ALR of the kinetic coefficient, is expressed as follows:

$$\begin{cases} A_1 = 2(\ln|v_x| - \ln|v_z|) \\ A_2 = 2(\ln|v_y| - \ln|v_z|) \end{cases}$$

Finally, the matrix expression of this system is:

$$\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} 2 & 0 & -2 \\ 0 & 2 & -2 \end{pmatrix} \begin{pmatrix} \ln|v_x| \\ \ln|v_y| \\ \ln|v_z| \end{pmatrix}$$

4.2 Centred logratio transformation *CLR*

The CLR transformation is defined as:

$$CLR : S_{n+1} \to \mathbb{R}^{n+1}$$
$$(x_1, \ldots, x_{n+1}) \to \left(\ln \frac{x_1}{g(x)}, \ln \frac{x_2}{g(x)}, \ldots, \ln \frac{x_{n+1}}{g(x)} \right)$$

In this expression, $g(x)$ is the geometric mean of the components.

In our case:

$$CLR : S_3 \to \mathbb{R}^3$$
$$\vec{E} \to CLR(\vec{E}) = \left(\ln \frac{E_1}{g(E)}, \ln \frac{E_2}{g(E)}, \ln \frac{E_3}{g(E)} \right) = (C_1, C_2, C_3)$$

In this expression $g(E)$ is the geometric mean of the wind kinetic coefficient:

$$g(E) = \sqrt[3]{E_1 E_2 E_3} = \left(\left(\frac{v_x}{v}\right)^2 \left(\frac{v_y}{v}\right)^2 \left(\frac{v_z}{v}\right)^2 \right)^{1/3} = \frac{v_x^{2/3} v_y^{2/3} v_z^{2/3}}{v^2}$$

Then,

$$CLR(\vec{E}) = \left(\ln \frac{\left(v_x/v\right)^2}{g(E)}, \ln \frac{\left(v_y/v\right)^2}{g(E)}, \ln \frac{\left(v_z/v\right)^2}{g(E)} \right)$$

Simplifying

$$CLR(\vec{E}) = \left(\ln \frac{v_x^2}{v_x^{2/3} v_y^{2/3} v_z^{2/3}}, \ln \frac{v_y^2}{v_x^{2/3} v_y^{2/3} v_z^{2/3}}, \ln \frac{v_z^2}{v_x^{2/3} v_y^{2/3} v_z^{2/3}} \right)$$

$$CLR(\vec{E}) = \left(\ln \frac{v_x^{4/3}}{v_y^{2/3} v_z^{2/3}}, \ln \frac{v_y^{4/3}}{v_x^{2/3} v_z^{2/3}}, \ln \frac{v_z^{4/3}}{v_x^{2/3} v_y^{2/3}} \right)$$

Finally, we have:

$$\begin{cases} C_1 = \frac{4}{3}\ln|v_x| - \frac{2}{3}\ln|v_y| - \frac{2}{3}\ln|v_z| \\ C_2 = -\frac{2}{3}\ln|v_x| + \frac{4}{3}\ln|v_y| - \frac{2}{3}\ln|v_z| \\ C_3 = -\frac{2}{3}\ln|v_x| - \frac{2}{3}\ln|v_y| + \frac{4}{3}\ln|v_z| \end{cases}$$

The matrix expression of this linear system is:

$$\begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix} = \begin{pmatrix} 4/3 & -2/3 & -2/3 \\ -2/3 & 4/3 & -2/3 \\ -2/3 & -2/3 & 4/3 \end{pmatrix} \begin{pmatrix} \ln|v_x| \\ \ln|v_y| \\ \ln|v_z| \end{pmatrix}$$

4.3 Isometric logratio transformation *ILR*

The ILR transformation is defined as:

$$ILR : S_{n+1} \to H \subset \mathbb{R}^{n+1}$$
$$\left(x_1, ..., x_{n+1}\right) \to \left(\sqrt{\frac{1}{2}} \ln \frac{x_1}{x_2}, \sqrt{\frac{2}{3}} \ln \frac{\sqrt{x_1 x_2}}{x_3}, ..., \sqrt{\frac{n}{n+1}} \ln \frac{g(x_1, ..., x_n)}{x_{n+1}} \right)$$

In this case, we have:

$$ILR : S_3 \to H \subset \mathbb{R}^3$$
$$\vec{E} \ \to ILR(\vec{E}) = \left(\sqrt{\frac{1}{2}} \ln \frac{E_1}{E_2}, \sqrt{\frac{2}{3}} \ln \frac{\sqrt{E_1 E_2}}{E_3} \right) = \left(I_1, I_2\right)$$

$$ILR(\vec{E}) = \left(\sqrt{\frac{1}{2}} \ln \frac{\left(v_x/v\right)^2}{\left(v_y/v\right)^2}, \sqrt{\frac{2}{3}} \ln \frac{\sqrt{\left(v_x/v\right)^2 \left(v_y/v\right)^2}}{\left(v_z/v\right)^2} \right)$$

$$ILR(\vec{E}) = \left(\sqrt{\frac{1}{2}} \ln \frac{v_x^2}{v_y^2}, \sqrt{\frac{2}{3}} \ln \left| \frac{v_x v_y}{v_z^2} \right| \right)$$

$$ILR(\vec{E}) = \left(\sqrt{\frac{1}{2}} (2\ln|v_x| - 2\ln|v_y|), \sqrt{\frac{2}{3}} (\ln|v_x| + \ln|v_y| - 2\ln|v_z|) \right)$$

Consequently, the ILR components are:

$$\begin{cases} I_1 = \sqrt{\frac{1}{2}} (2\ln|v_x| - 2\ln|v_y|) \\ I_2 = \sqrt{\frac{2}{3}} (\ln|v_x| + \ln|v_y| - 2\ln|v_z|) \end{cases}$$

The matrix expression of this linear system is:

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 2/\sqrt{2} & -2/\sqrt{2} & 0 \\ \sqrt{2/3} & \sqrt{2/3} & -2\sqrt{2/3} \end{pmatrix} \begin{pmatrix} \ln|v_x| \\ \ln|v_y| \\ \ln|v_z| \end{pmatrix}$$

5. Conclusions

1. Wind components can be considered as compositional variables, considering the squares of wind vector components.
2. From this consideration, we can define a wind kinetic coefficient through which is possible to analyse the atmospheric dynamics according to the methodology based on compositional data.
3. Usual compositional data transformations applied to wind kinetic coefficient can be interpreted from energetic point of view in the wind studies.

6. References

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