Weibull parameters distribution fitting in the surface wind layer

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ABSTRACT
Aeolian energy is currently a viable alternative to traditional energies. Several studies have been made to design and to build Aeolian parks for power provision of small, medium or great levels of consumption. In order to determine the viability of parks' locations, statistical studies in situ are made measuring the wind speed during long periods of time with expensive material to the approximated height of the nose of the aero generator.

This paper aims to establish a mathematical relation, based on empirical data, between parameters of the Weibull distribution at different heights in the atmospheric superficial layer, to reduce the price of cost and to improve decision making.

KEYWORDS: Aeolian energy, Weibull distribution, curve fitting.

1. Introduction

Aeolian energy is currently an important item in energetic world complementary to traditional power source (petroleum, gas, atomic…) as this is a kind of clean and renewable energy. Its importance grows with increasing surroundings preservation consciousness and also with the aim to make a sustainable power system. To install wind power generators it is very important to know wind regime, that is the variables that describe it (direction, intensity, speed, etc), at surface and also in altitude. For a level h since soil surface, average wind velocity follows a Weibull distribution. This work shows a methodology to have the functional expression dependency of Weibull distribution parameters with level h.

As it is stated, the average in uniform periods of time of module m of wind vector velocity follows a Weibull distribution. Equation (1) shows its density function. k is known as shape parameter (adimensional) and c scale parameter (in m/s). Parameters k, c are found through an adjusting process from experimental data; when $k = 2$, Weibull distribution is known also as Rayleigh distribution.

$$f(m) = \frac{k}{c}\left(\frac{m}{c}\right)^{k-1} \exp\left[-\left(\frac{m}{c}\right)^{k}\right],\ m \geq 0 \tag{1}$$

2. Data set

The experimental data set used consists of wind velocity measures at different level: 3, 6, 10, 13, 20, 32, 50, 100 metres. The wind tower was installed in Valladolid (Spain) and measures were taken in September 1998. Data show average wind velocity (module) in 5 minutes' interval, taken 20 times per second. So, that means a total of 4848 data that correspond to a 17 days period. This great bulk of data makes results statistically reliable for this purpose. Table 1 shows data main descriptive statistics. Descriptive data analysis is completed in Figure 1, which shows relative frequency distribution data histograms; those histograms have the same scale in both axes to allow a better comparison and understanding of height distribution evolution.

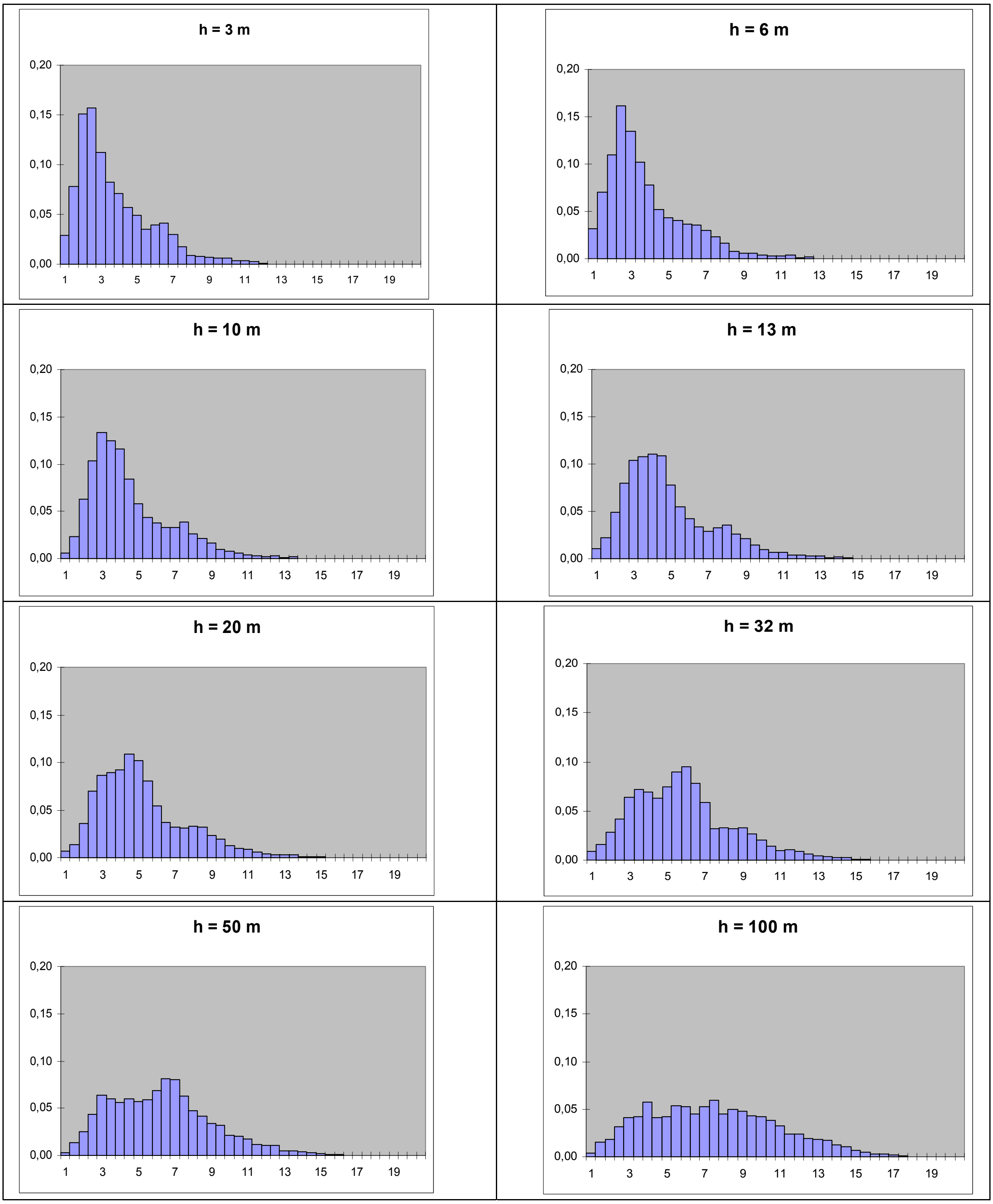

Figure 1. Relative frequency distribution data histograms

h (m)	3	6	10	13	20	32	50	100
Mean	3,50	3,57	4,31	4,61	4,96	5,59	6,04	7,23
Standard Deviation	2,08	2,06	2,23	2,35	2,41	2,60	2,77	3,42
Skewness	1,22	1,24	1,16	1,07	1,01	0,72	0,56	0,37
Minimum	0,06	0,08	0,41	0,24	0,10	0,05	0,40	0,05
Maximum	12,59	12,74	14,04	14,71	15,15	15,79	16,02	19,27

Table 1. Data main descriptive statistics

3. Weibull distribution parameters estimation

As it is said, the average module of wind vector velocity follows a Weibull distribution, which parameters have to be estimated form experimental data. So, Weibull distribution parameters have been estimated from histograms and data statistics analysis. Methodology applied to do so is maximum likelihood. Results are shown in Table 2.

***h* (m)**	3	6	10	13	20	32	50	100
k	1,811	1,853	2,063	2,088	2,191	2,278	2,328	2,252
***c* (m/s)**	0,083	0,075	0,038	0,032	0,023	0,015	0,011	0,009

Table 2. Weibull distribution parameters

4. Weibull distribution parameters functional fitting

Until now, as far as we know, Weibull distribution parameters were expressed numerically, like in Table 2. As those parameters depend on height it seems reasonable to have their functional expression. To obtain an approach to this function that expresses the relation of the parameters with height *h*, we have fit some functions to the values of Table 2. Several functional expressions have been tried: exponential, logarithmic, potential and polynomial. The best results correspond to potential and logarithmic functions. These adjustments are shown in Table 3 and in Figure 2.

	Function	*a*	*b*	R^2
50 m	$k = a + b \ln h$	1,5659	0,2015	0,9643
	$k = ah^b$	1,6138	0,0979	0,9583
100 m	$k = a + b \ln h$	1,6676	0,1551	0,8476
	$k = ah^b$	1,6956	0,0753	0,8425

Table 3(a). Function fit for shape parameter

	Function	*a*	*b*	R^2
50 m	$c = a + b \ln h$	0,1119	-0,0279	0,9069
	$c = ah^b$	0,2271	-0,7661	0,9694
100 m	$c = a + b \ln h$	0,1011	-0,0230	0,8579
	$c = ah^b$	0,2019	-0,7124	0,9696

Table 3(b). Function fit for scale parameter

50 m 100 m

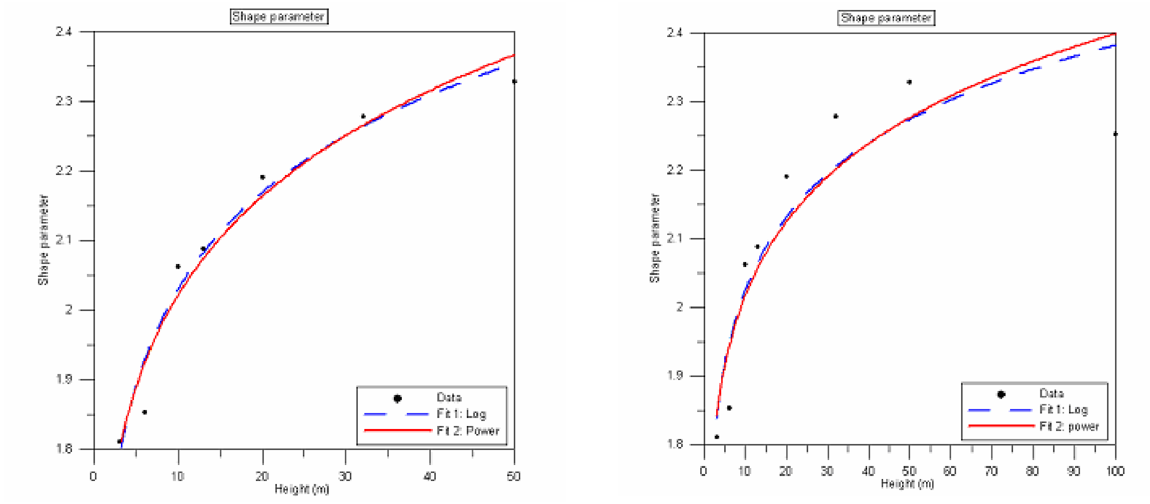

Figure 2(a). Curve functional fitting for shape parameter.

50 m 100 m

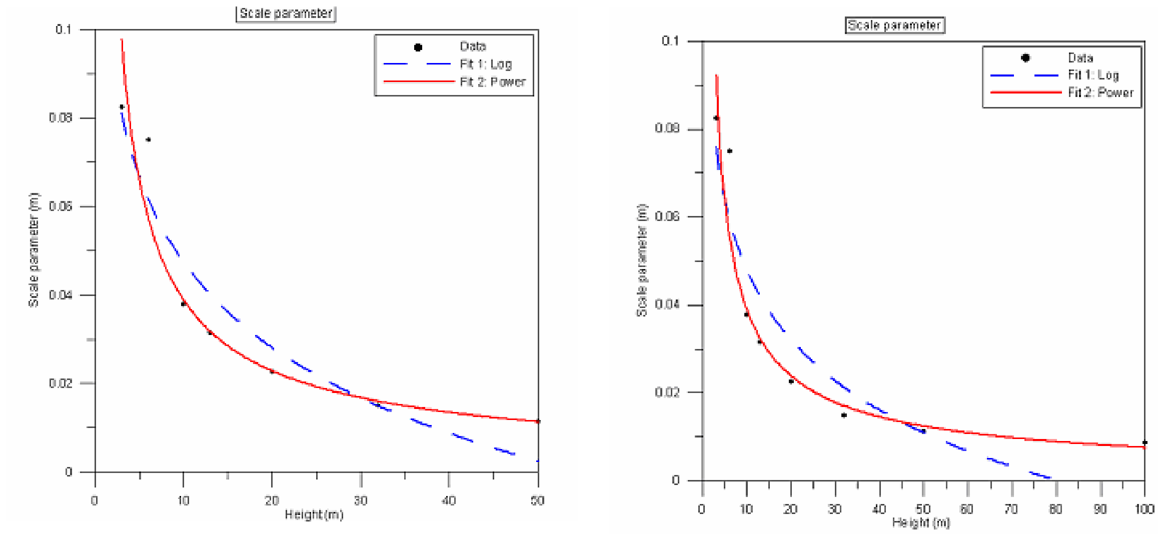

Figure 2(b). Curve functional fitting for scale parameter.

Conclusions

1) Functional expressions of Weibull distribution parameters as height functions can be obtained from reliable data sets.
2) The best function found is a logarithmic one for shape parameter and a power one for scale parameter.

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